

Luenberger Indicator and Directions of Measurement: A Bottoms-up Approach with an Empirical Illustration to German Saving Banks

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Abstract. The Luenberger productivity indicator applies directional distance functions which allow to specifying in what direction (i.e. direction of measurement) the operating units will be evaluated. In the presence of a change in the direction of measurement, the standard components of the existing Luenberger productivity indicator may provide values which are not compatible with reality. In order to eliminate this pitfall, the so-called bottoms-up approach is used to revisit the definition of the indicator and its components. We start with a list of selected sources of productivity change, namely efficiency change, technical change and direction change, then examine the best possible way of measuring each of the sources and combine them to derive a new measure of productivity change. The proposed indicator will be illustrated by means of an empirical application to a panel of 417 German saving banks over the time period 2006-2012. The example explains how the proposed approach is able to properly measure efficiency change, technical change and direction change. The results also provide conclusive evidence about the effect of the change in direction of measurement on the results of the productivity over time in a centralized management scenario.

Keywords: Data Envelopment Analysis (DEA); productivity measurement; direction change; banking

JEL Classification C44, C67, D24

1. Introduction

Among the different indices for measuring productivity changes of decision making units (DMUs) over time, Malmquist indices have commonly been used by researchers and practitioners in various environments. Examples include the health sector (e.g., see Kirigia et al. 2007; Chowdhury et al. 2011), the electricity industry (e.g., see Tovar et al. 2011; Aghdam 2011), telecommunications (e.g., see Lam and Shiu 2010; Hisali and Yawe 2011), the water industry (e.g., see Corton and Berg 2009; Portela et al. 2011), agriculture (e.g., see Kao 2010; Xu 2012), transportation (e.g., see Gitto and Mancuso 2012; Pires and Fernandes 2012), the banking industry (e.g., see Asmild et al. 2004; Portela and Thanassoulis 2010) and others.

Caves et al. (1982) introduced the earliest type of the Malmquist index and showed how the change in productivity experienced by an operating unit can be measured over time. Nischimizu and Page (1982) identified technological change and change in technical efficiency as two components of productivity change over time. Färe et al. (1992) used data envelopment analysis (DEA), proposed by Farrell (1957) and developed by Charnes et al. (1978), as mathematical programming-based methodology to measure the Malmquist productivity index. In the same paper, they also showed how the Malmquist index can be exhibited as the product of technical change and efficiency change components (i.e. FGLR decomposition of the Malmquist index). After this seminal work, there have been a considerable number of studies in the literature about the framework (see, e.g. Berg et al. 1992; Shestalova 2003; Pastor and Lovell 2005; Pastor et al. 2011), decomposition (see, e.g., Färe et al. 1994; Ray and Desli 1997; Wheelock and Wilson 1999; Gilbert and Wilson 1998; Grifell-Tatje and Lovell 1999), and computation (see, e.g., Chen 2003; Grifell-Tatje et al. 1998; Portela et al. 2004) of the Malmquist index.

Since the introduction of the primal Malmquist index by Färe et al. (1992), one of the limitations often faced by researchers in measuring this index has been to choose either an input- or an output-oriented perspective. The reason is that the Malmquist index requires a choice to be made between an input distance function and an output distance function yielding the input and output Malmquist productivity indices, respectively. In contrast, many practical situations suggest to combine both views, i.e. input-saving and output-expanding scenarios have to be taken into account simultaneously. In order to overcome this limitation, Chambers et al. (1996) introduced the Luenberger productivity indicator (hereafter named Luenberger indicator) for measuring productivity changes over time. The authors showed that this indicator, which has an additive structure rather than multiplicative, contains the input/output Malmquist productivity indices as its special cases. Motivated by FGLR decomposition of the Malmquist index, they also describe how

the Luenberger indicator can be decomposed into technical change (shift in the frontier of the benchmark technology) and changes in technical efficiency (change in the individual initiatives and activities) as two components of productivity change over time.

The Luenberger indicator applies directional distance functions which allow to specifying in what direction (i.e. direction of measurement) the operating units will be evaluated (see, e.g., Färe and Grosskopf 2000). Within this framework, the performance of a unit is characterized by measuring the distance to the boundary of the benchmark technology along the predetermined direction of measurement, i.e. a directed distance is defined. This property provides the possibility to work with a multidirectional productivity analysis in a way that a desired structure (e.g., central management's preference) concerning the potential improvement of inputs and outputs can be incorporated. It also enables to deal with special structures of input/output data when measuring productivity changes over time. Examples are the Luenberger-type indicators to measure environmentally sensitive productivity growth where some outputs are undesirable (see, e.g., Chung et al. 1997) and to measure productivity change under negative data (see, e.g., Portela et al. 2010). Among other advantages (see, e.g., Chambers et al. 1998), the above-described property of directional distance functions has made the Luenberger indicator an important managerial tool which can facilitate decision making and control in performance management systems.

A review of the studies focusing on multiple time period analysis leads to the conclusion that not only the shape and the characteristics of the benchmark technology can change (e.g., due to policy directives, the competitive situation and economic conditions) but also the direction of measurement. Among different situations in which the direction of measurement is likely to change over time, we address the scenario that a centralized management exists which supervises the operating units. In such cases, the centralized management of the organization is often responsible, e.g., for coordinating decision making within the group, determining strategic directions and making general policy decisions as well as monitoring the activities of the operating units. Within this scenario, some variables are controlled by the central management not only to promote efficiency and effectiveness but also to improve the level of learning, coordination and motivation among the operating units (Bogetoft and Otto 2011). Possible examples concern organizations with operating units like bank branches, pharmacy stores, university departments, police stations etc. (different perspectives on centralized assessment of operating units by DEA can be found, e.g., in Athanassopoulos 1995; Li and Ng 1995; Lozano and Villa 2004; Cook and Zhu 2007; Asmild et al. 2009; Ahn et al. 2012; Fang 2013).

In cases like that, a preferred direction of measurement can be determined with regard to the corporate strategy and overall goals of the organization. This direction is usually beyond the control of local managers and may change over time. Thereby, the responsible employees in the operating units are often rewarded on the basis of the results from the performance measurement system (for a detailed discussion of this issue see, e.g., Langfield-Smith 1997; Nudurupati et al. 2011). In such a context, any change in the direction of measurement can force the operating units to adapt their local variables (e.g., local strategy, scale of operation etc.) in order to avoid their productivity to be affected over time. Accordingly, apart from efficiency change (change in the individual initiatives and activities) and technical change (shift in the frontier of the benchmark technology) as two standard drivers of productivity change, any regress or progress in the productivity of a unit may also be explained by considering the change in the direction of measurement.

As it will be shown, the existing two-way decomposition of the Luenberger indicator is unable to distinguish between the shift in the frontier of the benchmark technology and the change in the direction of measurement. Consequently, in the presence of a change in the direction of measurement, the standard components of the Luenberger indicator may provide values which are not compatible with reality. This pitfall has not been identified or solved so far in previous studies where the direction of measurement is addressed and, among others, is defined as the mean values (see, e.g., Park and Weber 2006) or the ideal point (see, e.g., Portela and Thanassoulis 2010) of the data in each time period. Against this background, we revisit the Luenberger indicator and its components in order to remedy the outlined pitfall. Using the bottoms-up approach suggested by Balk (2001) we start with a list of selected sources of productivity change, examine the best possible way of measuring each of these sources and combine them to derive a new measure of productivity change. The new indicator will not only properly measure efficiency change and technical change components, but is also able to capture the degree to which predetermined directions of measurement affect the productivity of units over time.

The paper proceeds as follows: Section 2 presents an overview of the process of efficiency measurement by means of directional distance functions. It will also be shown how performance, which comprises magnitude and direction, can systematically be affected by different specifications of both benchmark technology and direction of measurement. In section 3, it will be investigated why the existing two-way decomposition of the Luenberger indicator is unable to properly measure productivity change in the presence of a change in the direction of measurement. The proposed Luenberger indicator and the corresponding components – namely efficiency change, technical change and direction change – will be introduced and described in Section 4. The mathematical aspects of the proposed indicator will also be investigated. Section 5 analyzes the proposed

Luenberger indicator and its advantages on the basis of an empirical illustration to a panel of 417 German saving banks over the time period 2006-2012. Section 6 concludes the paper with a summary and an outlook on future research opportunities.

2. Benchmark technology and directional distance function

Suppose that there exist n DMUs in t ($t = 1, \dots, T$) time periods. Let $X_j^t = (x_{1j}^t, x_{2j}^t, \dots, x_{mj}^t) \in \mathfrak{R}_+^m$ and $Y_j^t = (y_{1j}^t, y_{2j}^t, \dots, y_{sj}^t) \in \mathfrak{R}_+^s$ be non-zero vectors which quantify the level of inputs and outputs of DMU _{j} in period t . The benchmark technology, which is defined as the set of all feasible combinations of input and output quantities in t , is usually shown as:

$$T^t = \left\{ (X^t, Y^t) \in \mathfrak{R}_+^m \times \mathfrak{R}_+^s \mid X^t \text{ can produce } Y^t \right\}. \quad (1)$$

In terms of properties satisfied by each benchmark technology, T^t can be characterized precisely by applying desired mathematical axioms such as free disposability, returns to scale, convexity etc. (see, e.g., Charnes et al. 1978; Banker et al. 1984). Throughout the paper, without loss of generality (see, e.g., Färe et al. 1994), we assume that each benchmark technology satisfies the following axioms:

1. (Non-emptiness). The observed $(X_j^t, Y_j^t) \in T^t$, $j = 1, \dots, n$.
2. (Free disposability). If $(X, Y) \in T^t$, $X' \geq X$, $Y' \leq Y$, then $(X', Y') \in T^t$.
3. (Constant returns to scale). If $(X, Y) \in T^t$, then $(\alpha X, \alpha Y) \in T^t$ for all $\alpha \geq 0$.
4. (Convexity). If (X, Y) and (\tilde{X}, \tilde{Y}) , then $\lambda(X, Y) + (1 - \lambda)(\tilde{X}, \tilde{Y}) \in T^t$ for any $\lambda \in [0, 1]$.
5. (Minimum extrapolation). T^t is the smallest set which satisfies axioms 1 to 4.

The benchmark technology in time period t can now be specified as follows:

$$T^t = \left\{ (X^t, Y^t) \in \mathfrak{R}_+^m \times \mathfrak{R}_+^s \mid X^t \geq \sum_{j=1}^n \lambda_j^t x_{ij}^t, \quad Y^t \leq \sum_{j=1}^n \lambda_j^t y_{rj}^t; \quad \lambda_j^t \geq 0; \quad j = 1, \dots, n \right\}. \quad (2)$$

Following Chambers et al. (1998), the directional distance function which simultaneously seeks to expand the outputs and contract the inputs in time period t can be defined as:

$$\hat{D}^u(X^t, Y^t, d_x, d_y) = \sup \left\{ \delta : (X^t, Y^t) + \delta(-d_x, d_y) \in T^t \right\}, \quad (3)$$

where $\vec{d} = (-d_x, d_y)$ defines a directional vector so that $d_x = (d_{x1}, d_{x2}, \dots, d_{xm}) \in \mathfrak{R}_+^m$ and $d_y = (d_{y1}, d_{y2}, \dots, d_{ys}) \in \mathfrak{R}_+^s$. This direction allows us to work with a multidirectional efficiency analysis by which we can incorporate a desired structure (e.g., central management's preference) over the potential improvement of inputs and outputs. Detailed properties of the directional distance function can be found in Chambers et al. (1996; 1998).

Figure 1 illustrates the process of measurement by means of a simple case of production process in which a single input is used to produce a single output.

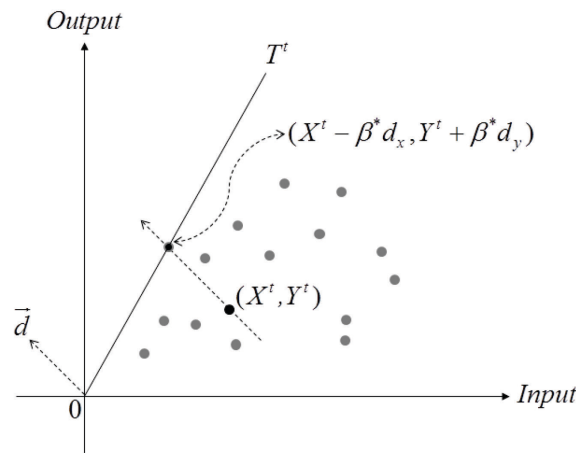


Figure 1. Benchmark technology and directional distance function.

According to definition (3), given a directional vector \vec{d} , (X^t, Y^t) is projected onto the boundary of the technology T^t at $(X^t - \delta^* d_x, Y^t + \delta^* d_y)$, where $\delta^* = \hat{D}^u(X^t, Y^t, d_x, d_y)$. On this basis, the value of the distance function δ^* depends not only on the characteristics of the benchmark technology T^t , but also on the direction of measurement \vec{d} as well as on the corresponding distance of the unit under evaluation from the frontier of the benchmark technology. In other words, the performance comprises both magnitude and direction and is systematically affected by different specifications of both the benchmark technology and the direction of measurement. Therefore, this can be considered as a primary motivation to distinguish between properties which characterize the benchmark technology and the choice of direction by which the performance is measured. The reason is that, on one hand, the performance is concerned by the shape of the benchmark technology which can be affected itself by regulations, competitive situations and economic conditions etc. On the other hand, it can be significantly oriented towards the direction of measurement determined with regard to the corporate strategy and overall goals of the organization. A detailed discussion of

the role of the benchmark technology in the process of efficiency measurement can be found, e.g., in Grosskopf (1986).

3. The Luenberger indicator and change in the direction of measurement

Suppose that an individual unit, DMU_p ($p = 1, \dots, n$), in time periods t and $t+1$ is represented by $DMU_p^t = (X_p^t, Y_p^t)$ and $DMU_p^{t+1} = (X_p^{t+1}, Y_p^{t+1})$, respectively. In order to measure the productivity change for this unit between the two time periods, the directional distance functions can be determined corresponding to either the first technology T^t or the second technology T^{t+1} as best-practice reference. Accordingly, Chambers et al. (1996) define the Luenberger indicator, here denoted by $LI(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t)$, as the arithmetic mean of the two measures of productivity change which are computed on the benchmark technologies t and $t+1$ as follows:

$$LI(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) = \frac{1}{2} \left\{ \left[\hat{D}^{t+1}(X_p^t, Y_p^t, d_x, d_y) - \hat{D}^{t+1}(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) \right] + \left[\hat{D}^t(X_p^t, Y_p^t, d_x, d_y) - \hat{D}^t(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) \right] \right\} \quad (4)$$

Furthermore, it has been shown that the Luenberger indicator can additively be decomposed into the following components (for further details see, e.g., Chambers et al. 1996):

$$\begin{aligned} \text{Efficiency Change (EC)} &= TE^t(X_p^t, Y_p^t, d_x, d_y) - TE^{t+1}(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) \\ &= \hat{D}^t(X_p^t, Y_p^t, d_x, d_y) - \hat{D}^{t+1}(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) \end{aligned} \quad (5)$$

$$\begin{aligned} \text{Technical Change (TC)} &= TC^{t+1,t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t, d_x, d_y) \\ &= \frac{1}{2} \left\{ \left[\hat{D}^{t+1}(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) - \hat{D}^t(X_p^{t+1}, Y_p^{t+1}, d_x, d_y) \right] \right. \\ &\quad \left. + \left[\hat{D}^{t+1}(X_p^t, Y_p^t, d_x, d_y) - \hat{D}^t(X_p^t, Y_p^t, d_x, d_y) \right] \right\} \end{aligned} \quad (6)$$

This decomposition reveals that the change in productivity can be affected by two components. The former is efficiency change (EC). It captures the change in the technical efficiency of the unit under consideration between time periods t and $t+1$. The latter is technical change (TC). It is computed by an arithmetic mean of the two basic technical changes which represent the change in the frontier of the benchmark technology between the two time periods.

If the value of the Luenberger indicator or any of its components is less than one, it denotes regress, while a value greater than one implies progress; a value of one indicates an unchanged situation. In

order to explain the computation details of the Luenberger indicator and its decomposition, Figure 2 is given in which DMU_p in periods t and $t+1$ is denoted by a and f , respectively.

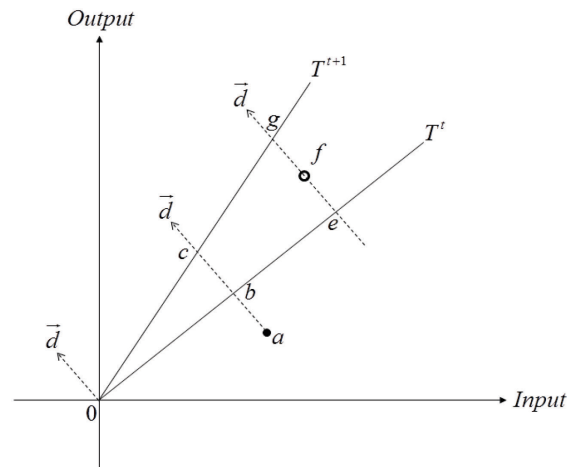


Figure 2. Example to illustrate the Luenberger indicator.

Consider the following terms which have been obtained in connection with Figure 2:

$$\begin{aligned} b &= a + D^{\frac{u}{u}}(a, d) \cdot d & e &= f + D^{\frac{u}{u}}(f, d) \cdot d \\ c &= a + D^{\frac{u}{u}}(a, d) \cdot d & g &= f + D^{\frac{u}{u}}(f, d) \cdot d \end{aligned} \quad (7)$$

By means of these notations, the two components of the Luenberger indicator defined in (5) and (6) are determined as follows:

$$\text{Efficiency Change}(EC) = (b - a) - (g - f) \quad (8)$$

$$\text{Technical Change}(TC) = \frac{1}{2} \{ (g - e) + (c - b) \} \quad (9)$$

According to (8), efficiency change indicates whether the unit under evaluation is closer to or further away from the boundary of the benchmark technology T^{t+1} compared to its situation in benchmark technology T^t . Technical change in (9), which is computed by the average distance between the two benchmark technologies, represents the change in the boundary of the technology over time. The two components are independent of each other: There can be technical change without efficiency change or efficiency change without technical change.

It should be noted that the two components here use the same direction of measurement $\frac{u}{u}$ in the computation process. Therefore, it is questionable whether these components can still provide the correct results in the presence of a change in the direction of measurement, e.g., where the mean of data is used as directional vector in each time period. In order to investigate this case, suppose a

particular example in which the direction of measurement changes over time. Let the corresponding directions of measurement in time periods t and $t+1$ be denoted by \vec{d}^t and \vec{d}^{t+1} , respectively. Furthermore, let assume that the benchmark technology remains unchanged, i.e. $T^t = T^{t+1} (= T^{UC})$, and that the unit under consideration is inefficient in either periods of time, i.e. it does not operate on the boundary of the technology. A graphical example of this case is shown in Figure 3.

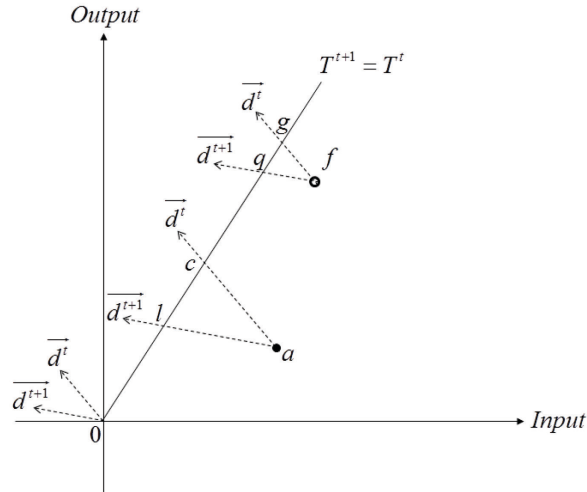


Figure 3. The Luenberger indicator and a change in the direction of measurement.

By adapting the notations introduced in (7) with regard to the case depicted in Figure 3, we obtain the following terms:

$$\begin{aligned} c &= a + D^{UC}(a, \vec{d}^t) \cdot \vec{d}^t & l &= a + D^{UC}(a, \vec{d}^{t+1}) \cdot \vec{d}^{t+1} \\ g &= f + D^{UC}(f, \vec{d}^t) \cdot \vec{d}^t & q &= f + D^{UC}(f, \vec{d}^{t+1}) \cdot \vec{d}^{t+1} \end{aligned} \quad (9)$$

By means of the formulas in (5) and (6), the *EC* and *TC* components are now determined as follows:

$$\text{Efficiency Change}(EC) = (c - a) - (q - f) \quad (10)$$

$$\text{Technical Change}(TC) = \frac{1}{2} \{ (q - g) + (l - c) \} \quad (11)$$

From (10), we can see that the *EC* component can still capture a change in the efficiency of the unit under consideration in such a way that efficiency has been computed relative to both the present technology as well as the present direction of measurement in each time period (yet to be investigated in greater detail in Section 4). Furthermore, as can be taken from (11), the presence of any inefficiency for the unit under evaluation results in a non-zero value for the *TC* component. This means that the boundary of the technology in the region this unit operates should have moved.

However, this is obviously not the case, what can be observed from the figure which is based on the primal assumption that $T^t = T^{t+1} (= T^{UC})$. It must therefore be concluded that the current TC component is unable to properly characterize technological progress/regress as change in the boundary of the technology. The reason is that this component does not distinguish between the change in frontier technology and another important factor which can capture the change in the direction of measurement. In the presence of such a kind of change, the TC component and accordingly the entire decomposition may provide values which are not compatible with what has been experienced in reality.

In order to eliminate the depicted pitfall, the so-called bottoms-up approach is used in the following to revisit the definition of the aforementioned components in the presence of change in the direction of measurement. More precisely, we start with a list of selected sources of productivity change, then examine the best possible way of measuring each of the sources and combine them to derive a measure of productivity change. As it will be shown, the resulting new indicator properly measures efficiency change as well as technical change components and is able to capture the degree to which predetermined directions of measurement affect the productivity of units over time.

4. The proposed Luenberger indicator

4.1 Notations

Consider n DMUs observed in time period t ($t = 1, \dots, T$). We use the same notations for the level of inputs and outputs as well as the same assumptions for the benchmark technologies as introduced in Section 2. We consider the case that not only the benchmark technology can change over time but also the direction of measurement. Therefore, we use the following modified notation of the directional distance function:

$$D_{d^t}^t(X^t, Y^t) = \sup \left\{ \delta : (X^t, Y^t) + \delta(-d_x^t, d_y^t) \in T^t \right\}, \quad (12)$$

where $\overline{d^t} = (-d_x^t, d_y^t)$ defines a directional vector in time period t ($t = 1, \dots, T$) so that $d_x^t = (d_{x1}^t, d_{x2}^t, \dots, d_{xm}^t) \in \mathfrak{R}_+^m$ and $d_y^t = (d_{y1}^t, d_{y2}^t, \dots, d_{ys}^t) \in \mathfrak{R}_+^s$. Figure 4 shows a graphical example with two time periods t and $t+1$, accordingly two benchmark technologies T^t and T^{t+1} as well as two directions of measurement $\overline{d^t}$ and $\overline{d^{t+1}}$.

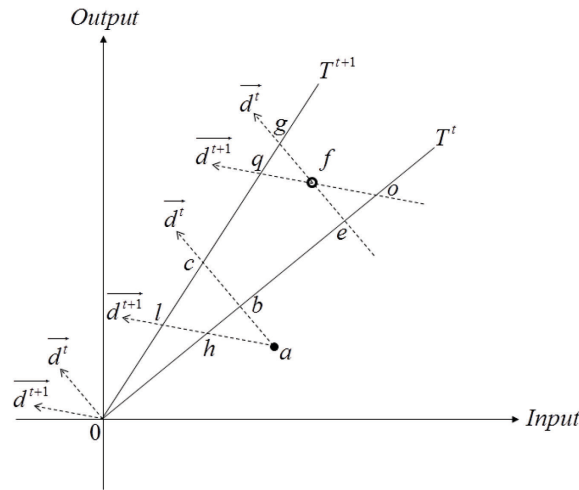


Figure 4. Example for modelling a change in the direction of measurement.

By means of our new notation for the directional distance function introduced in (12), we obtain the following terms in connection with Figure 4 for DMU_p , which has been denoted by a and f in periods t and $t+1$, respectively.

$$\begin{aligned}
 b &= a + \bar{D}_{d^t}^{t,t}(a, d^t) \cdot d^t & h &= a + \bar{D}_{d^{t+1}}^{t,t}(a, d^{t+1}) \cdot d^{t+1} \\
 e &= f + \bar{D}_{d^t}^{t,t}(f, d^t) \cdot d^t & o &= f + \bar{D}_{d^{t+1}}^{t,t}(f, d^{t+1}) \cdot d^{t+1} \\
 c &= a + \bar{D}_{d^t}^{t,t+1}(a, d^t) \cdot d^t & l &= a + \bar{D}_{d^{t+1}}^{t,t+1}(a, d^{t+1}) \cdot d^{t+1} \\
 g &= f + \bar{D}_{d^t}^{t,t+1}(f, d^t) \cdot d^t & q &= f + \bar{D}_{d^{t+1}}^{t,t+1}(f, d^{t+1}) \cdot d^{t+1}
 \end{aligned} \tag{13}$$

4.2 Efficiency change component

The inefficiency of a DMU has been defined as the maximum amount that one can contract the inputs and/or expand the outputs along the predetermined direction of measurement (see Figure 1 in Section 2). We therefore define the efficiency measure of the unit under evaluation in time period t with respect to the observed benchmark technology T^t as well as the direction of measurement d^t which has been selected in this period (see Figure 5a). According to (12), the corresponding directional distance function is denoted by $\bar{D}_{d^t}^{t,t}(X^t, Y^t)$. Likewise, the efficiency measure of this unit in time period $t+1$, which is denoted by $\bar{D}_{d^{t+1}}^{t,t+1}(X^{t+1}, Y^{t+1})$, corresponds to the observed benchmark technology T^{t+1} and the direction of measurement d^{t+1} selected in time period $t+1$ (see Figure 5b).

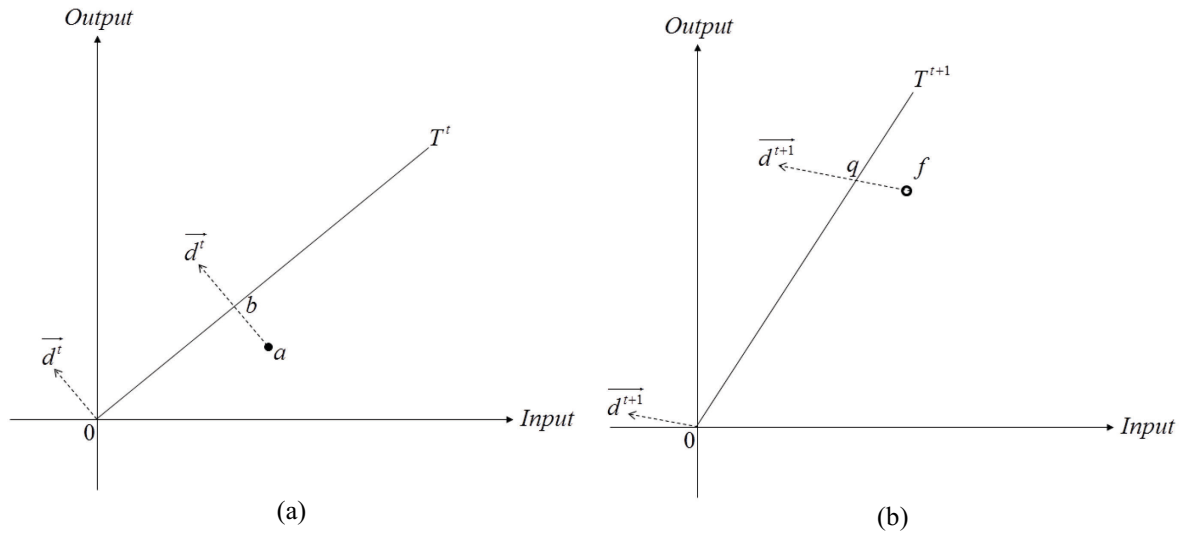


Figure 5. Cases (a) and (b) with focus on change in the efficiency.

We define the efficiency change (*EC*) component as the difference in these two measures as follows:

$$\begin{aligned} \text{Efficiency Change}(EC) &= TE_{d^t}^{T^t}(X_p^t, Y_p^t) - TE_{d^{t+1}}^{T^{t+1}}(X_p^{t+1}, Y_p^{t+1}) \\ &= \hat{D}_{d^t}^{T^t}(X_p^t, Y_p^t) - \hat{D}_{d^{t+1}}^{T^{t+1}}(X_p^{t+1}, Y_p^{t+1}) \end{aligned} \quad (14)$$

By means of the terms given in (13) and the above formula, the efficiency change for DMU_{*p*} in Figure 4 can now be determined as follows:

$$\text{Efficiency Change}(EC) = (b - a) - (q - f) \quad (15)$$

From (15), we can observe that the efficiency change indicates whether the unit under evaluation is closer to or further away from the boundary of the benchmark technology T^{t+1} (where the direction of measurement $\overline{d^{t+1}}$ has also been imposed) compared to its situation by taking into account the benchmark technology T^t (coupled with the direction of measurement $\overline{d^t}$).

4.3 Technical change component

According to the widely used definition of technical change (TC), this component should reflect an improvement or a deterioration in the performance resulting from the shift of the technology frontier over time (see, e.g., Färe et al. 1992; Grosskopf 2003; Aparicio et al. 2013). In other words, there exists technical change over two time periods t and $t+1$, if the boundary of the technology in the region the unit under evaluation operates moves and this movement affects the performance.

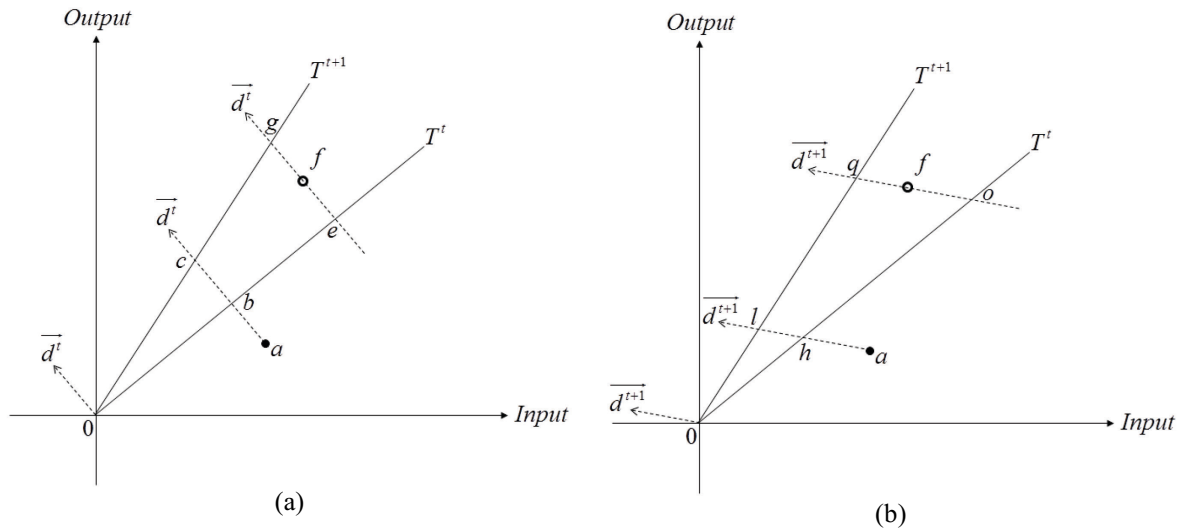


Figure 6. Cases (a) and (b) with focus on the shift of the technology frontier.

Taking into account the direction of measurement \vec{d}^t , we can measure the shift of the boundary of the technology between two time periods t and $t+1$ by means of the following technical change term:

$$TC_{d^t}^{T^{t+1}, T^t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) = \frac{1}{2} \left\{ \left[\hat{D}_{d^t}^{t+1}(X_p^{t+1}, Y_p^{t+1}) - \hat{D}_{d^t}^t(X_p^{t+1}, Y_p^{t+1}) \right] + \left[\hat{D}_{d^t}^{t+1}(X_p^t, Y_p^t) - \hat{D}_{d^t}^t(X_p^t, Y_p^t) \right] \right\} \quad (16)$$

This is the standard technical change component defined earlier in (6) which is based on the directional vector \vec{d}^t for the measurement of the corresponding distance functions (see Figure 6a). However, the technical change component in (16) is a function of the choice of directions of measurement (i.e. \vec{d}^t and \vec{d}^{t+1}), yielding the two following measures of technical change: $TC_{d^t}^{T^{t+1}, T^t}$ and $TC_{d^{t+1}}^{T^{t+1}, T^t}$ (cf. Figure 6b and 6a). Hence, in order to avoid an arbitrary choice among them, both measures should be combined. In this respect, we propose to use their arithmetic mean which can be defined as follows:

$$\begin{aligned}
\text{Technical Change}(TC) &= \frac{1}{2} \left\{ TC_{d^t}^{T^{t+1}, T^t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) + TC_{d^{t+1}}^{T^{t+1}, T^t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) \right\} \\
&= \frac{1}{4} \left\{ \left[\hat{B}_{d^t}^{t+1}(X_p^{t+1}, Y_p^{t+1}) - \hat{B}_{d^t}^t(X_p^{t+1}, Y_p^{t+1}) \right] + \left[\hat{B}_{d^t}^{t+1}(X_p^t, Y_p^t) - \hat{B}_{d^t}^t(X_p^t, Y_p^t) \right] \right. \\
&\quad \left. \left[\hat{B}_{d^{t+1}}^{t+1}(X_p^{t+1}, Y_p^{t+1}) - \hat{B}_{d^{t+1}}^t(X_p^{t+1}, Y_p^{t+1}) \right] + \left[\hat{B}_{d^{t+1}}^{t+1}(X_p^t, Y_p^t) - \hat{B}_{d^{t+1}}^t(X_p^t, Y_p^t) \right] \right\} \quad (17)
\end{aligned}$$

By means of the terms given in (13) and the above formula, the technical change for DMU_p in Figure 4 can now be determined as follows:

$$\text{Technical Change}(TC) = \frac{1}{4} \{ (g - e) + (c - b) + (q - o) + (l - h) \} \quad (18)$$

As can be seen in (18), the framework to measure the technical change component has been equipped with the two directions of measurement so that it is able to properly capture the shift of the technology frontier between the two periods of time.

Supposing an unchanged direction of measurement $\bar{d}^t = \bar{d}^{t+1} (= \bar{d}^{UC})$ in t and $t+1$, the indicator in (17) will collapse to the traditional one suggested by Chambers et al. (1996). Furthermore, the proposed indicator of technical change always provides a value of zero for situations in which $T^t = T^{t+1} (= T^{UC})$, as it is the case for the DMU_p under evaluation in Section 3 (see Figure 3). This desirable property is easy to see from (17).

4.4 Direction change component

We introduce the concept of direction change in order to measure the effect of predetermined directions of measurement on the results of productivity over time. There exists direction change (DC) over two time periods t and $t+1$, if the direction of measurement in the region the unit under evaluation operates change and this change affects the performance. Accordingly, the DC component will reflect improvement or deterioration in the performance resulting from the change in the direction of measurement over time.

On the basis of the benchmark technology T^t , we can measure the change in the direction of measurement between two time periods t and $t+1$ by means of the following direction change term:

$$\begin{aligned}
DC_{d^{t+1}, d^t}^{T^t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) \\
= \frac{1}{2} \left\{ \left[\bar{D}_{d^{t+1}}^{T^t}(X_p^t, Y_p^t) - \bar{D}_{d^t}^{T^t}(X_p^t, Y_p^t) \right] + \left[\bar{D}_{d^{t+1}}^{T^t}(X_p^{t+1}, Y_p^{t+1}) - \bar{D}_{d^t}^{T^t}(X_p^{t+1}, Y_p^{t+1}) \right] \right\} \quad (19)
\end{aligned}$$

The first term inside the curly bracket evaluates the effect of change in the direction of measurement on the unit under consideration in period t , whereas the second term captures this effect on the unit in period $t+1$. As can be seen from (19), we define DC as the average change in the direction of measurement from period t to period $t+1$ where benchmark technology T^t has been used for the measurement of the corresponding distance functions (see Figure 7a).

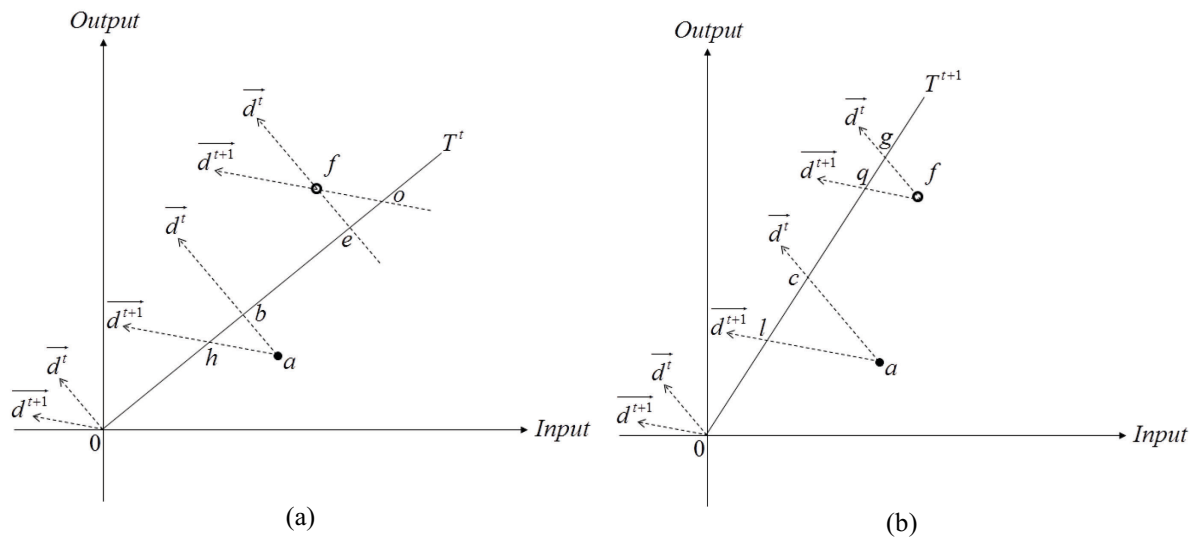


Figure 7. Cases (a) and (b) with focus on the change in the direction of measurement.

The basic direction change in (19) is a function of the choice of the benchmark technologies (i.e. T^t and T^{t+1}), yielding the two following measures of direction change: $DC_{d^{t+1}, d^t}^{T^t}$ and $DC_{d^{t+1}, d^t}^{T^{t+1}}$ (cf. Figure 7a and 7b). Hence, the recommended measure of direction change component is defined as the arithmetic mean of these two measures as follows:

$$\begin{aligned}
\text{Direction Change}(DC) &= \frac{1}{2} \left\{ DC_{d^{t+1}, d^t}^{T^t}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) + DC_{d^{t+1}, d^t}^{T^{t+1}}(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) \right\} \\
&= \frac{1}{4} \left\{ \left[\bar{D}_{d^{t+1}}^{T^t}(X_p^t, Y_p^t) - \bar{D}_{d^t}^{T^t}(X_p^t, Y_p^t) \right] + \left[\bar{D}_{d^{t+1}}^{T^t}(X_p^{t+1}, Y_p^{t+1}) - \bar{D}_{d^t}^{T^t}(X_p^{t+1}, Y_p^{t+1}) \right] \right. \\
&\quad \left. \left[\bar{D}_{d^{t+1}}^{T^{t+1}}(X_p^t, Y_p^t) - \bar{D}_{d^t}^{T^{t+1}}(X_p^t, Y_p^t) \right] + \left[\bar{D}_{d^{t+1}}^{T^{t+1}}(X_p^{t+1}, Y_p^{t+1}) - \bar{D}_{d^t}^{T^{t+1}}(X_p^{t+1}, Y_p^{t+1}) \right] \right\} \quad (20)
\end{aligned}$$

With the terms given in (13) and the above formula, the direction change for DMU_p in Figure 4 can now be determined as follows:

$$Direction\ Change(DC) = \frac{1}{4} \{ (h-b) + (o-e) + (l-c) + (q-g) \} \quad (21)$$

The direction change indicator in (21) is based on the two benchmark technologies; it allows us to properly capturing the contribution of change in the direction of the measurement to productivity change between the two periods of time.

Supposing an unchanged benchmark technology for t and $t+1$, the proposed indicator in (20) will collapse to the basic one in (19). Then, the proposed direction change component computed for the unit under evaluation in Section 3 becomes

$$Direction\ Change(DC) = \frac{1}{2} \{ (q-g) + (l-c) \} \quad (22)$$

quantifying the contribution of change in the direction of the measurement in the presence of the assumption that $T^t = T^{t+1} (= T^{UC})$. It is also easy to see from (20) that for situations in which $\overline{d}^t = \overline{d}^{t+1} (= \overline{d}^{UC})$, the proposed indicator of direction change always provides a value of zero which is compatible with what has been imposed as a primal assumption.

4.5 The productivity change indicator

Having defined three sources of productivity change, namely efficiency change (EC), technical change (TC) and direction change (DC), the following new Luenberger indicator (LI) can combine these components to measure productivity change over two time periods t and $t+1$:

$$LI(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) = EC + TC + DC \quad (23)$$

The components can be measured by means of (14), (17) and (20), respectively. After substitutions and algebraic manipulations, the following expression for the productivity change indicator can be derived:

$$LI(X_p^{t+1}, Y_p^{t+1}, X_p^t, Y_p^t) = \frac{1}{2} \left\{ \left[\hat{B}_{d^{t+1}}^{t+1}(X_p^t, Y_p^t) - \hat{B}_{d^{t+1}}^{t+1}(X_p^{t+1}, Y_p^{t+1}) \right] + \left[\hat{B}_{d^t}^t(X_p^t, Y_p^t) - \hat{B}_{d^t}^t(X_p^{t+1}, Y_p^{t+1}) \right] \right\} \quad (24)$$

The result in (24) is very similar to the standard definition of the Luenberger indicator in (4). Compared to (4), the directional distance functions involved are computed on the basis of the benchmark technologies associated with the direction of measurement which has been selected at

the time. In other words, the direction of measurement \overline{d}^k is considered as an element of T^k ($k = t, t+1$). Consequently, the standard Luenberger indicator in (4) coincides with the proposed measure of productivity change in (24) where \overline{d}^k (resp. \overline{d}^{k+1}) coupled with T^k (resp. T^{k+1}) is used in the computation of the indicator. However, the two-way decomposition and the corresponding components derived by (4) are not identical with those proposed in this section. It is clear that the *EC* and *TC* components of the two approaches will give the same values if the direction of measurement remains unchanged over time: the direction change in (20) becomes zero and the efficiency change and technical change components in (14) and (17) will collapse to the standard ones given in (5) and (6), respectively.

4.6 Mathematical computations

It can be taken from (14), (17), (20) and (24) that the proposed Luenberger indicator as well as the corresponding components for DMU_p ($p = 1, \dots, n$) over time periods t and $t+1$ can be determined by the directional distance functions $\hat{D}_{d^\beta}^{\alpha}(X_p^t, Y_p^t)$, $\alpha, \beta = t, t+1$, and $\hat{D}_{d^\beta}^{\alpha}(X_p^{t+1}, Y_p^{t+1})$, $\alpha, \beta = t, t+1$. With respect to the definition of the directional distance function in (12), these functions can be computed by means of the following formulas:

$$\hat{D}_{d^\beta}^{\alpha}(X_p^t, Y_p^t) = \sup \left\{ \delta_\beta^\alpha : (X^t, Y^t) + \delta_\beta^\alpha (-d_x^\beta, d_y^\beta) \in T^\alpha \right\}, \quad \alpha, \beta = t, t+1 \quad (25)$$

$$\hat{D}_{d^\beta}^{\alpha}(X_p^{t+1}, Y_p^{t+1}) = \sup \left\{ \delta_\beta^\alpha : (X^{t+1}, Y^{t+1}) + \delta_\beta^\alpha (-d_x^\beta, d_y^\beta) \in T^\alpha \right\}, \quad \alpha, \beta = t, t+1 \quad (26)$$

where $\overline{d}^\beta = (-d_x^\beta, d_y^\beta)$, $\beta = t, t+1$ defines the directional vectors in time periods t and $t+1$. In addition, the benchmark technologies T^α , $\alpha = t, t+1$ can be specified according to (2) as follows:

$$T^\alpha = \left\{ (X, Y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s \left| X \geq \sum_{j=1}^n \lambda_j^\alpha x_{ij}^\alpha, \quad Y \leq \sum_{j=1}^n \lambda_j^\alpha y_{rj}^\alpha; \quad \lambda_j^\alpha \geq 0; \quad j = 1, \dots, n \right. \right\}. \quad (27)$$

Considering (25) - (27), the mathematical formulations for the determination of the above distance functions are now straightforward. Substituting (27) in (25) and (26), the corresponding distance functions for DMU_p ($p = 1, \dots, n$) in time periods t and $t+1$ can be determined by means of the following linear programming problems:

$$D_{d^\beta}^{\beta^\alpha}(X_p^t, Y_p^t) = \max \left\{ \delta_\beta^\alpha \left| \begin{array}{l} \sum_{j=1}^n \lambda_j^\alpha x_{ij}^\alpha \leq x_{ip}^t - \delta_\beta^\alpha d_x^\beta, \quad i = 1, \dots, m; \\ \sum_{j=1}^n \lambda_j^\alpha y_{rj}^\alpha \geq y_{rp}^t + \delta_\beta^\alpha d_y^\beta, \quad r = 1, \dots, s; \\ \lambda_j^\alpha \geq 0, \quad \delta_\beta^\alpha \text{ free in sign.} \end{array} \right. \right\} \quad (28)$$

$$D_{d^\beta}^{\beta^\alpha}(X_p^{t+1}, Y_p^{t+1}) = \max \left\{ \delta_\beta^\alpha \left| \begin{array}{l} \sum_{j=1}^n \lambda_j^\alpha x_{ij}^\alpha \leq x_{ip}^{t+1} - \delta_\beta^\alpha d_x^\beta, \quad i = 1, \dots, m; \\ \sum_{j=1}^n \lambda_j^\alpha y_{rj}^\alpha \geq y_{rp}^{t+1} + \delta_\beta^\alpha d_y^\beta, \quad r = 1, \dots, s; \\ \lambda_j^\alpha \geq 0, \quad \delta_\beta^\alpha \text{ free in sign.} \end{array} \right. \right\} \quad (29)$$

where s is the number of outputs; m is the number of inputs; n is the number of DMUs evaluated; x_{ij}^α is the value of input i ($i = 1, \dots, m$) and y_{rj}^α is the value of output r ($r = 1, \dots, s$) for DMU $_j$ ($j = 1, \dots, n$); λ_j^α is the intensity variable attached to DMU $_j$ ($j = 1, \dots, n$); δ_β^α is an unrestricted variable whose optimal value determines a respective directional distance function for DMU $_p$ ($p = 1, \dots, n$).

5. An empirical illustration to German saving banks

In order to illustrate how the proposed indicator measures the productivity change over time, we analyze a panel of 417 German saving banks (i.e. $n = 417$ DMUs) over the time period 2006-2012 (i.e. $t = 2006, \dots, 2012$). As only a few banks have been deleted from the analysis because of inadequate information, the sample of 417 consists of 97% of all German saving banks comprising three inputs ($m = 3$) and two outputs ($s = 2$) during the time period under consideration. In the following the characteristics of the data set along a brief overview of the structure of German savings banks are investigated. Moreover, the parameter values of the mathematical programming problems (see section 4.6), which are used for computing the proposed Luenberger indicator and its corresponding components, will be specified.

The group of German savings banks represents not only the largest banking sector in Germany but also in the world. These banks, which operate under a common trade brand *Sparkasse*, are essentially credit institutions under public law. Their responsible government departments (but not owners) are the local authorities (e.g. municipalities and regional associations) in the regions a particular saving bank is situated. In this context, saving banks are considered as so-called non-profit institutions whose aims are supporting their municipalities and regional associations in their obligation to facilitate economic development, regional policy and social as well as cultural programs (for further details about the structure of German saving banks see, e.g., Vitols 1995; Simpson 2013).

German savings banks are not a consolidated group and operate independently in their respective regions. Each bank is locally administrated by its own management board which is comprised of

banking professionals and qualified members. The management board is responsible for the day-to-day conduct of the business and reporting to a supervisory board of representatives of the customers, employees and the regional association/council. Furthermore, saving banks are also controlled centrally by the German Savings Banks Association (*Deutscher Sparkassen- und Giroverband, DSGV*) which is the umbrella organization responsible for coordinating decision making within the group, determining strategic directions, making general policy decisions and monitoring the activities of the banks to ensure effective and efficient operation with low risk. (see, e.g., dsgv.de; Simpson 2013).

In the banking literature, productivity measurement and improvement using DEA-based productivity change indicators/indices have been addressed in many theoretical and application-oriented studies. An extensive literature review can be found, e.g., in Chen and Yang (2011) as well as in Paradi and Zhu (2013). In order to measure productivity, input and output factors of banks' activities must be determined. Two popular approaches have been widely used by researchers, the production approach and the intermediation approach (Asmild et al. 2004). The production approach treats banks as producers of products and services such as loans and deposits using labor, fixed assets and operating expenses. In the intermediation approach, banks are considered as financial intermediaries, which collect monetary funds from savers/investors and transpose these funds into further investments. According to these views and based on the data we had access to, we specified the following inputs and outputs:

- Input #1 (x_1): number of employees,
- Input #2 (x_2): fixed assets,
- Input #3 (x_3): total non-interest expenses,
- Output #1 (y_1): total customer deposits,
- Output #2 (y_2): total loans.

The selected input and output data have been extracted from the Bankscope database. Descriptive statistics of the three inputs and two outputs over the time period 2006-2012 are given in Table 1.

Table 1. Descriptive statistics of the inputs and outputs used in this study.

	<i>Number of employees(x_1)</i>						
	2006	2007	2008	2009	2010	2011	2012
Min	39	39	42	39	40	38	39
Max	4785	5328	5434	5547	5622	5724	5891
Mean	525.77	523.34	530.03	533.04	532.53	531.52	531.82
St. Dev.	492.46	498.29	498.04	502.91	508.95	510.86	515.13
	<i>Fixed assets(x_2)</i>						
	2006	2007	2008	2009	2010	2011	2012
Min	400	500	500	500	700	600	500
Max	163300	174900	176900	171600	167500	178800	221500
Mean	27690.65	26605.52	25903.84	25343.17	24971.70	24526.86	24386.33
St. Dev.	27525.77	26365.59	26007.50	26190.09	26085.73	25813.39	26138.33
	<i>Total non-interest expenses(x_3)</i>						
	2006	2007	2008	2009	2010	2011	2012
Min	2800	3000	2900	3100	3200	3100	3400
Max	650700	660300	658400	694700	747100	813500	842300
Mean	43877.94	45000.72	44462.59	45810.79	45253.72	47389.21	49041.25
St. Dev.	52075.44	52929.78	52277.99	53552.82	54881.49	67232.55	70802.30
	<i>Total customer deposits(y_1)</i>						
	2006	2007	2008	2009	2010	2011	2012
Min	83300	86100	89500	94400	95800	95600	97900
Max	22534400	22805200	25543300	26884300	27122300	27392700	27977400
Mean	1418720.86	1478100.96	1551620.38	1606601.44	1658254.68	1700399.52	1751694.96
St. Dev.	1736168.56	1799788.41	1942096.79	2017807.13	2059021.12	2100355.65	2177764.19
	<i>Total loans(y_2)</i>						
	2006	2007	2008	2009	2010	2011	2012
Min	69400	70000	61800	58900	57100	59300	63900
Max	21938900	21355600	22667500	24213400	26165500	27730900	29865400
Mean	1286707.19	1299539.81	1338154.68	1361285.61	1405734.53	1465370.74	1513901.92
St. Dev.	1653111.77	1636937.48	1727723.88	1784676.43	1873045.80	1962518.98	2067717.46

The amounts of fixed assets, total non-interest expenses, total customer deposits and total loans are given in thousand Euro.

In order to compute the proposed Luenberger indicator and its corresponding components, we need to specify in what direction the banks will be evaluated. This allows a multidirectional analysis which incorporates a desired structure (i.e. DSGV's preference) over the potential improvement of inputs and outputs for saving banks. Since the primary goal of this empirical application is to illustrate the proposed indicator, we consider the simplest case in which a directional vector is defined based on an ideal point in period t ($t = 2006, \dots, 2012$) as follows:

$$\mathbf{u}^t = (-d'_{x_1}, -d'_{x_2}, -d'_{x_3}; d'_{y_1}, d'_{y_2}); \quad d'_{x_i} = \min \left\{ x_{ij}^t \right\}_{j=1}^{417}, \quad i = 1, 2, 3; \quad d'_{y_r} = \max \left\{ y_{rj}^t \right\}_{j=1}^{417}, \quad r = 1, 2 \quad (30)$$

On this basis, the directional distance functions will measure how a bank could increase its outputs and simultaneously reduce its inputs in the direction which is constructed on the basis of a

hypothetical unit with maximum outputs and minimum inputs in each time period. Values of the components of the above directional vectors can also be obtained from Table 1. It should be noted that the idea of the ideal point has also been used in different contexts for the measurement of efficiency and productivity change (see, e.g., Färe et al. 2004; Portela et al., 2004; Portela and Thanassoulis 2010).

In order to show different features of the proposed indicator, two scenarios are considered. In scenario #1, the directional vectors remain unchanged in some periods of time, while in scenario #2, the ideal point of the data of each particular year is used as directional vector. Table 2 depicts the results: The direction of measurement in scenario #1 is changed only in the second, fourth and the last adjacent periods; in scenario #2, the direction of measurement changes in each period, since the ideal point of data changes over time.

Table 2. Two scenarios for selecting directions of measurement.

	<i>Directions of measurement</i>											
	2006 - 2007		2007 - 2008		2008 - 2009		2009 - 2010		2010 - 2011		2011 - 2012	
Scenario #1	\overline{d}^{2006}	\overline{d}^{2006}	\overline{d}^{2006}	\overline{d}^{2008}	\overline{d}^{2008}	\overline{d}^{2008}	\overline{d}^{2008}	\overline{d}^{2010}	\overline{d}^{2010}	\overline{d}^{2010}	\overline{d}^{2010}	\overline{d}^{2012}
Scenario #2	\overline{d}^{2006}	\overline{d}^{2007}	\overline{d}^{2007}	\overline{d}^{2008}	\overline{d}^{2008}	\overline{d}^{2009}	\overline{d}^{2009}	\overline{d}^{2010}	\overline{d}^{2010}	\overline{d}^{2011}	\overline{d}^{2011}	\overline{d}^{2012}

According to the analysis put forward in section 4, the proposed Luenberger indicator and its components for each bank (i.e. DMU_p ; $p = 1, \dots, 417$) in each pair of adjacent time periods (i.e. over two time periods t and $t+1$) are determined by eight distance functions. As it has been shown in section 4.6, the required distance functions can be computed by means of the mathematical programming problems (28) and (29) whose parameters α and β are now specified as follows:

1. $(\alpha, \beta) = (t, t)$ for determining $\overline{D}_{d^t}^{t,t}(X_p^t, Y_p^t)$ by means of model (28).
2. $(\alpha, \beta) = (t+1, t)$ for determining $\overline{D}_{d^t}^{t,t+1}(X_p^t, Y_p^t)$ by means of model (28).
3. $(\alpha, \beta) = (t, t+1)$ for determining $\overline{D}_{d^{t+1}}^{t,t}(X_p^t, Y_p^t)$ by means of model (28).
4. $(\alpha, \beta) = (t+1, t+1)$ for determining $\overline{D}_{d^{t+1}}^{t,t+1}(X_p^t, Y_p^t)$ by means of model (28).
5. $(\alpha, \beta) = (t, t)$ for determining $\overline{D}_{d^t}^{t,t+1}(X_p^{t+1}, Y_p^{t+1})$ by means of model (29).
6. $(\alpha, \beta) = (t+1, t)$ for determining $\overline{D}_{d^t}^{t,t+1}(X_p^{t+1}, Y_p^{t+1})$ by means of model (29).
7. $(\alpha, \beta) = (t, t+1)$ for determining $\overline{D}_{d^{t+1}}^{t,t+1}(X_p^{t+1}, Y_p^{t+1})$ by means of model (29).
8. $(\alpha, \beta) = (t+1, t+1)$ for determining $\overline{D}_{d^{t+1}}^{t,t+1}(X_p^{t+1}, Y_p^{t+1})$ by means of model (29).

The resulting mathematical programming problems have been encoded in AIMMS, version 3.13. For each bank ($p = 1, \dots, 417$) in each scenario (i.e. scenarios #1 and #2) and in each pair of adjacent

time periods (i.e. 2006-07, 2007-08, 2008-09, 2009-10, 2010-11, 2011-12) the above eight distance functions have been calculated. Thus, in total ($417 \times 2 \times 6 \times 8 =$) 40032 linear programming problems have been solved. The results have subsequently been used to determine the proposed Luenberger indicator and its components for each bank in each scenario and in each pair of adjacent time periods as follows:

- Efficiency change (*EC*) component has been determined by applying formula (14) on the basis of the results of the distance functions in (1) and (8),
- Technical change (*TC*) component has been determined by applying formula (17) on the basis of the results of the distance functions in (1), (2), (4), (5), (6), (7) and (8),
- Direction change (*DC*) component has been determined by applying formula (20) on the basis of the results of the distance functions in (1)-(8),
- The Luenberger indicator (*LI*) has been determined by applying formula (24) on the basis of the results of the distance functions in (1), (4), (5) and (8).

The corresponding results of the Luenberger indicator and its components have been summarized in Table 3. As can be seen, the mean value of *LI* for each of the six adjacent periods (hereafter adj-period) in both scenarios #1 and #2 is non-negative, signifying that productivity has never decreased during the whole period analyzed. However, a significant fluctuation of the productivity improvement can be observed around the third adj-period (2008-2009), e.g. for scenario #1 with 0.4% in 2007-2008, 0.0% in 2008-2009, and 0.4% in 2009-2010. This is the time period that encompasses the world financial crisis. Subsequently, a downward trend follows, starting from the fourth adj-period (2009-2010) to the end of the time horizon; e.g., productivity in scenario #2 is falling from 0.3% in 2009-2010 to 0.0% in 2011-2012.

Table 3 shows how the Luenberger indicator has additively been decomposed into the three sources of productivity change theoretically discussed in Section 4: efficiency change (*EC*: contribution of change in the individual initiatives and activities), technical change (*TC*: contribution of shift in the frontier of the benchmark technology) and direction change (*DC*: contribution of change in direction of measurement). The results of the *EC* and *TC* components in scenario #1 reveal the same pattern as the corresponding results in scenario #2. The first two adj-periods show non-negative values of these components, followed by adj-periods with both declines and improvements. In contrast to that, the results of the direction change component in the two scenarios report some discrepancies, i.e. the adj-periods with progress/regress are not always the same in the two scenarios. This will now be further investigated.

Table 3. Results of the proposed Luenberger indicator and its corresponding components.

		Scenario #1				Scenario #2			
		EC	TC	DC	LI	EC	TC	DC	LI
Adj-period 1: 2006-2007	Min	-0.066	-0.018	0.000	-0.062	-0.065	-0.018	-0.002	-0.061
	Max	0.096	0.062	0.000	0.091	0.095	0.062	0.004	0.089
	Mean	0.002	0.000	0.000	0.002	0.002	0.000	0.000	0.002
	StDev	0.011	0.004	0.000	0.011	0.011	0.004	0.001	0.011
Adj-period 2: 2007-2008	Min	-0.045	-0.021	-0.020	-0.042	-0.045	-0.021	-0.018	-0.044
	Max	0.076	0.108	0.005	0.113	0.074	0.108	0.004	0.112
	Mean	0.006	0.001	-0.003	0.004	0.006	0.001	-0.003	0.004
	StDev	0.011	0.006	0.003	0.012	0.011	0.006	0.003	0.012
Adj-period 3: 2008-2009	Min	-0.063	-0.034	0.000	-0.075	-0.060	-0.033	-0.008	-0.072
	Max	0.068	0.102	0.000	0.102	0.073	0.099	0.003	0.102
	Mean	-0.006	0.006	0.000	0.000	-0.005	0.006	-0.001	0.000
	StDev	0.010	0.009	0.000	0.010	0.010	0.009	0.001	0.010
Adj-period 4: 2009-2010	Min	-0.079	-0.030	-0.346	-0.077	-0.046	-0.029	-0.006	-0.063
	Max	0.047	0.430	0.074	0.084	0.043	0.075	0.004	0.079
	Mean	0.005	-0.002	0.001	0.004	0.006	-0.002	-0.001	0.003
	StDev	0.012	0.022	0.019	0.009	0.008	0.006	0.001	0.008
Adj-period 5: 2010-2011	Min	-0.028	-0.049	0.000	-0.031	-0.026	-0.049	-0.004	-0.030
	Max	0.065	0.083	0.000	0.083	0.066	0.081	0.002	0.083
	Mean	0.005	-0.004	0.000	0.001	0.005	-0.004	0.000	0.001
	StDev	0.009	0.008	0.000	0.007	0.010	0.008	0.001	0.007
Adj-period 6: 2011-2012	Min	-0.090	-0.002	-0.011	-0.036	-0.091	-0.002	-0.006	-0.035
	Max	0.024	0.119	0.007	0.112	0.024	0.117	0.004	0.106
	Mean	-0.008	0.010	-0.002	0.000	-0.009	0.010	-0.001	0.000
	StDev	0.012	0.013	0.002	0.008	0.012	0.012	0.001	0.008

Considering scenario #1, the direction change component in the first, third and fifth adj-periods is zero which is not surprising, since the direction of measurement have been kept unchanged in the mentioned periods as a primary setting (see Table 2). With respect to the theoretical arguments put forward in Section 3.4, in this case not only the direction change component formula becomes zero, but also the efficiency change and technical change components collapse to the traditional ones by Chambers et al. (1996). Accordingly, the results for the behavior of the Luenberger indicator can here be explained by the *EC* and *TC* components only. For example, the mean value of the efficiency change component in the third adj-period is -0.6%, implying that the average efficiency has declined from 2008 to 2009. On the other hand, the technical change component reports a positive growth of 0.6% for this period. The two effects compensate each other, resulting in the Luenberger indicator value of 0.0%.

Compared to the third adj-period of scenario #1, the pattern of the efficiency and technical change of the fourth adj-period is reversed (*EC*: 0.5%; *TC*: -0.2%). Moreover, the productivity change in this period is also affected by a direction change (*DC*: 0.1%). This leads to an average value of 0.4% for the Luenberger indicator, signifying that there was a productivity increase from 2009 to

2010. As can be seen in scenario #2 for the same adj-period, the result of the technical change component is identical to the one obtained in scenario #1. However, the efficiency change component in the second scenario is 0.1% higher than in the first scenario. In addition, the direction change component signals a negative value of -0.1% in scenario #2, which is reversed compared to its value in scenario #1. This is due to the fact that the productivity change indicators of the two scenarios apply different directional vectors for the computation of the directional distance functions (cf. Table 2).

Scenario #2 reveals another aspect of the impact of the change in direction of measurement on the results of the productivity over time. During the periods analyzed, the direction change component reports negative values with the exception in the first (2006-2007) and fifth (2009-2010) adj-periods whose mean values of the direction change components are zero. Looking at the fifth period, e.g., the direction of measurement has been changed from 2010 to 2011 as a primary setting (see Table 2). Although the effect of this change on the results of the productivity varies between (Min:) -0.4% and (Max:) 0.2%, its mean value amounts to zero. This means that the average productivity change of 0.1% in this period can mostly be explained by considering the effect of the other components, i.e. of *EC* and *TC*: the amount of the technical change component in this period is -0.4% on average which captures a negative shift in the frontier of the benchmark technology from 2010 to 2011; however, the average efficiency has changed positively over this period by a value of 0.5% which results to the positive rate of growth of 0.1% in productivity.

It should be noted that the inefficiency of a unit has been defined as the maximum expansion in outputs and/or contraction in inputs along the predetermined direction of measurement. Therefore, change in efficiency as well as in the direction component will be zero for a bank which has been efficient in time periods t and $t+1$. In other words, such a bank not only remains a best practice unit but also any change in direction of measurement does not affect its productivity. A closer look at the results in general in both scenarios shows that four banks have been efficient in all time periods. Consequently, the direction change component for these banks is zero in all periods; compared to other, inefficient banks, that their performance has been less sensitive to changes in the direction of measurement over the selected periods.

6. Conclusions and Outlook on Future Research

The Luenberger indicator applies directional distance functions which allow to specifying in what direction (i.e. direction of measurement) the operating units will be evaluated. Within this framework, the performance of a unit is characterized by measuring the distance to the boundary

of the benchmark technology along the predetermined direction of measurement, i.e. a directed distance is defined. Arising from a series of practical cases, in multiple time period analysis not only the shape and the characteristics of the benchmark technology may change (e.g., due to policy directives, the competitive situation and economic conditions), but also the direction of measurement. However, the existing Luenberger indicator is unable to distinguish between these two sources of productivity changes. Consequently, in the presence of a change in the direction of measurement, the standard components of the indicator may provide values which are not compatible with reality.

In order to overcome the above-described problem, we have revisited the Luenberger indicator and its components. Making use of the bottoms-up approach, we started with a list of selected sources of productivity change, namely efficiency change, technical change and direction change. We then examined the best possible way of measuring each of these sources and combined them to derive a new measure of productivity change. The new indicator does not only measure efficiency change and technical change components in an appropriate way, but is also able to capture the degree to which predetermined directions of measurement affect the productivity of units over time.

The proposed framework is suitable especially for situations where some variables are controlled by the central management of an organization which supervises the operating units. In such cases, a preferred direction of measurement can be determined with regard to the corporate strategy and overall goals of the organization. This direction is usually beyond the control of local managers and may change over time. On this basis, any change in the direction of measurement can force the operating units to adapt their local variables (e.g., local strategy, scale of operation etc.) in order to avoid their productivity to be affected over time. Hence, the proposed framework can be used as a managerial control instrument to provide managers and policymakers with information to help them design better strategies aimed at achieving sustainable productivity growth.

In order to illustrate how the proposed Luenberger indicator measures the productivity change over time, a panel of 417 German saving banks over the time period 2006-2012 has been analyzed. In order to show different features of the proposed indicator, two scenarios for specifying the directions of measurement (i.e. DSGV's preference over the potential improvement of inputs and outputs) have been considered. The results demonstrated how the proposed approach is able to properly measure efficiency change and technical change, revealing effects of the change in direction of measurement on the results of the productivity over time. Moreover, a comparison of the results of both scenarios verified that the components of the Luenberger indicator may

change by different assumptions made about the directions of measurement over time. This highlights the fact that apart from efficiency change and technical change as two standard drivers of productivity change, any regress or progress in the productivity of a unit can be explained by considering the change in the direction of measurement.

The productivity indicators suggested in the literature have different properties and features. Depending on a specific situation with certain assumptions, it has to be decided which kind of indicator could be superior to the others. In this context, an interesting perspective for future research is to apply the proposed approach to other types of productivity change indicators which use an inter-temporal structure in their nature. An example is the sequential Malmquist index in which a sequential technology is formed from convex aggregation of observations in all periods up to the period under consideration (see, e.g., Shestalova 2003). Another example is the meta-frontier approach with an additional inter-temporal benchmark technology which is the convex union of some contemporaneous benchmark technologies (see, e.g., Battese et al. 2004).

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